3-5 Derivatives of the Trig Functions

Learning Objectives:

I can calculate the derivatives of the trig functions.

I can write the equation of the normal line to a curve.

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Derivatives of the Trig Functions

$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

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Ex1. Find the derivative

1.) $y = x^2 \cos x$

$y' = f \cdot g' + f' \cdot g$

$f(x) = x^2$ $\quad$ $g(x) = \cos x$

$f'(x) = 2x$ $\quad$ $g'(x) = -\sin x$

$y' = -x^2 \sin x + 2x \cos x$

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2.) $y = \sec x \cdot \tan x$

$y' = f' \cdot g + f \cdot g'$

$\sec x \cdot \tan^2 x + \sec^2 x \cdot \sec x$

$\sec x \cdot \tan^2 x + \sec^3 x$

$(\tan x)^2$

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3.) $f(x) = \cos x + \sin x - \tan x + x^2$

$f'(x) = -\sin x + \cos x - \sec^2 x + 2x$

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4.) $y = \frac{x^3}{\sin x}$ $\quad$ $f = x^3$ $\quad$ $f' = 3x^2$ $\quad$ $g = \sin x$ $\quad$ $g' = \cos x$

$y' = \frac{f' \cdot g - f \cdot g'}{g^2}$ $\quad$ $y' = \frac{3x^2 \cdot \sin x - x^3 \cdot \cos x}{\sin^2 x}$

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Opener

Non-Calculator

A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

$x' = 2t - 6 = 0$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$s' = 4\sin t - t = 4\sin 1.318 - 1.318$

$s'' = 4\cos t - 1 = 0$ Calculator $\cos t = \frac{1}{4}$ $t = \cos^{-1}\left(\frac{1}{4}\right) = 1.318$

A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{1t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) −5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

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Develop the rule for the derivative y=tanx using quotient rule

5.) $y = \tan x = \dfrac{\sin x}{\cos x}$

$f(x) = \sin x$ $g(x) = \cos x$

$f'(x) = \cos x$ $g'(x) = -\sin x$

$y' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$

$y' = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$y' = \dfrac{1}{\cos^2 x} = \sec^2 x$

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6.) $g(x) = \dfrac{\sec x}{x^2 + 1}$

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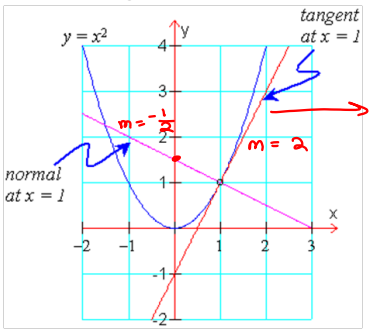
Tangent Line vs Normal Line



$y = 2x - 1$

$y = -\frac{1}{2}x + 1.5$

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Ex2a.) Find the equation of the tangent line to the curve $y = x^2 \sin x$ at $x = \dfrac{\pi}{3}$

$\left(\frac{\pi}{3}, \frac{\pi^2\sqrt{3}}{18}\right)$

$y' = x^2\cos x + 2x\sin x$

$y' = \left(\frac{\pi}{3}\right)^2 \cdot \cos\frac{\pi}{3} + 2\left(\frac{\pi}{3}\right)\sin\frac{\pi}{3} = \frac{\pi^2}{18} + \frac{2\pi\sqrt{3}}{6}$

$y - \frac{\pi^2\sqrt{3}}{18} = \left(\frac{\pi^2}{18} + \frac{2\pi\sqrt{3}}{6}\right)\left(x - \frac{\pi}{3}\right)$

b.) Find the equation of the normal line to the curvey=x²sinx at $x = \dfrac{\pi}{3}$

$y - \frac{\pi^2\sqrt{3}}{18} = \left(\frac{-18}{\pi^2 + 6\pi\sqrt{3}}\right)\left(x - \frac{\pi}{3}\right)$

$\frac{\pi^2}{18} + \frac{6\pi\sqrt{3}}{18}$

$\frac{\pi^2 + 6\pi\sqrt{3}}{18}$

(No Graphing Calculator)

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Homework

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